Résumé

This work deals with basic principles of arithmetic and calculus from the perspective of modern logic. The central line of the text examines the concept of real number or, more precisely, the concept of continuum in its development, beginning with Eudoxus, Descartes and Leibniz, through Bolzano, Cantor and Dedekind, to Frege, Brouwer, Hilbert and Wittgenstein. Every significant movement in the philosophy of mathematics (Platonism, formalism, intuitionism, logicism, constructivism, structuralism etc.) is covered, not abstractly, but against the background of particular results or phenomena of modern logic and mathematics. Among those mentioned are Cantor's diagonal argument, Dedekind's recursion theorem, Bolzano's intermediate value theorem, Russell's paradox along with other logical paradoxes, Zermelo's well-ordering theorem, Skolem's paradox, Hilbert's program, and Gödel's theorems. The book also contains the author's own original results relating to the untenability of a structuralistic (re)interpretation of Frege's logicism (in the neo-logicism of Wright and Hale) and a complex defense of semantic holism including its most radical inferentialist version. In mathematics, this is done against the background of the exegesis of Hilbert's works.

My goal, as author, was not just to delineate the evolution of significant arithmetical concepts, albeit discussed from a non-traditional analytical perspective. The classical issues of the philosophy of mathematics (number, continuum, infinity, abstraction etc.) are presented primarily as a motivation and key for solving problems relating to the philosophy of language (meaning, reference, truth, existence etc.) and to philosophy in general. The role of mathematics in this book, therefore, is only auxiliary, analogous to the roles it is given in the philosophies of Plato, Kant or Frege, all of whom considered mathematical knowledge to be a presupposition not subject to philosophical training. Due to its complicated, indirect relation to the world, mathematics for a philosopher is an important instrument with which to demonstrate the disputability of the presupposition that there exist immediate or natural facts standing free of the prior theory against which we discover them. Hence any facts we discover could, theoretically speaking, exist in an alternative guise. I try to elaborate on this by drawing on Bolzano's intermediate value theorem, reading it not in the usual way as an analytical proof of a self-evident fact (that a continuous function which takes opposite signs over a closed interval must include taking the value 0), but as a non-trivial proposal with which to define the continuum and, consequently, to provide a new proof and conceptual framework where the theorem is valid. In fact, this dependency of our observations on our theories is the idea behind Kant's Copernican turn and the basis of the finding that, as reasonable, self-conscious beings, we are not only products but also co-creators of reality.

This does not contradict the traditional view of mathematics as a prototype of stable and reliable knowledge. On the contrary, Parmenides' and Plato's distinction between the fluctuating, changeable world of senses on the one hand and the constant, eternal world of ideas on the other, is understood from the beginning as a basic philosophical distinction, the necessity of which is illustrated by mathematics. For instance, the discovery of irrational proportions (*alogoi logoi*) is something which, despite its intellectual inevitability, cannot be proved in a purely empirical fashion (since empirical measurements are always rational). Thus, Aristotle's attempt to eliminate Plato's ideas or concepts by their identification with their empirical extensions (e.g. the identification of the concept of cat with the set of actually existing cats, or the concept of circle with a set of — always imperfect — geometrical figures) is immediately dismissed as incorrect. It is only the view that the world of ideas is absolutely independent of us or that some people can or shall dwell in it, which is criticized. All distinctions (such as Plato's between the world of senses and the world of forms, Kant's between intuition and concept, or Frege's between object and concept) are only supporting moments in the process of our mastering and co-creating one single (or, if you wish, natural or everyday) world, in the spirit of Kant's thesis that real knowledge results only from both intuition and concept. Philosophy and logic, as treated in this book, are not disciplines that study or describe some other world, but merely the moderators of this communal process.